

Optimal Synthesis of a Cascade Refrigeration System of LNG Through MINLP Model for Pure Refrigerant Cycles

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Abstract: liquefied natural gas (LNG) is the most economical way of transporting natural gas (NG) over long distances. Liquefaction of NG by using vapor compression refrigeration system requires high operating and capital cost. Due to lack of systematic design methods for multistage refrigeration cycles, the conventional approaches of determining optimal cycle configuration are mostly based on trial-and-error and are very time consuming. Here a new Mixed Integer Non-Linear Programming (MINLP) model is introduced for selecting optimal synthesis of refrigeration system which would reduce both the OPEX and CAPEX of a LNG plant through Pinch technology. A superstructure of a complex refrigeration system is developed to select and optimize the key decision variables in refrigeration cycles (i.e. partition temperature, compression configuration, refrigeration features, refrigerant flow rate). This method reduces calculation time and increases design options that could be mounted over a multistage refrigeration cascade. Here a program (LNG-Pro) is developed that integrates the VBA, Refprop and Excel MINLP Solver to automate the methodology. Design of an LNG plant is described to illustrate advantages in adopting this method. Conceptual diagrams of pinch technology are shown to visualize the stepwise optimization procedure of compressor's shaftwork. The proposed cascade by this method reduces compressor shaftwork of an LNG cascade from 1255 Kj/Kg to 1141.9Kj/Kg.

Keywords: Cascade Refrigeration Cycle synthesis, Cryogenic, Liquefied Natural Gas, MINLP

1. Introduction

Natural gas (NG) is world's fastest growing energy commodity. Most NG reserves are offshore and away from demand sites. Liquefying NG to transport it to distances more than 3000km is the most economical way to export it by specially built ships to consuming market. A mega-scale LNG plant usually consumes 1188 kj energy to liquefy 1 kg of natural gas (Mortazavi et al., 2012). Therefore, energy saving is an immediate concern in LNG industry. NG is cooled from ambient temperature and liquefied to around -163°C

through costly refrigerating process. Such refrigeration system is equipped with world's largest compressors. At most 90% of the entering feed gas to a modern LNG plant is exported LNG and 10% of the gas is consumed to produce required shaftwork. High operating and capital cost of an LNG plant requires thorough studies on refrigeration cycle and optimal configuration of compressors to reduce cost.

Obtaining best refrigeration system configuration has become an intriguing due to its economic importance and the challenge of

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complex integration design between cascades. Barnés and King (1974) investigated a two-step approach to identify optimum cascade refrigeration systems for a limited number of promising choices of configurations and design parameters. Cheng and Mah (1980), proposed an interactive procedure for synthesizing refrigeration features. They introduced a heuristic method based on the average compression work to determine if additional intermediate stages are needed. B. Linnhoff and Engineers (1982) and B Linnhoff and Dhole (1989) used a set of qualitative guidelines based on pinch technology and exergy analysis for placing heat engines and heat pumps to minimize utility consumption. Mixed integer non-linear programming (MINLP) is the mathematical method to model existence or non-existence of different refrigeration features and compression configurations. Shelton and Grossmann (1986) introduced a discretized MINLP model on a refrigeration superstructure. This procedure can optimize many refrigeration features in pure refrigerant cycles. Faruque Hasan, Karimi, Alfadala, and Grootjans (2007) presented a MINLP approach on refrigeration systems to select the best network of an LNG plant. Castillo and Dorao (2012) discussed decision-making model for LNG processes based on game theory. Aspelund, Berstad, and Gundersen (2007) proposed a methodology based on pinch analysis to utilize pressure based exergy for sub-ambient processes. Mafi, Ghorbani, Amidpour, Mousavi Nayanian, & Salehi (2013a, 2013b) introduced a new method for systematic design of mixed refrigerant cycles with a given configuration that combines the advantages of thermodynamics approach and mathematical optimization. In this article the introduced method by Mafi et al (2013a, 2013b) is adopted

to find optimal synthesis of an LNG plant by mounting Mixed Integer Non-Linear Programming on it. The MINLP mathematical method is a powerful tool for decision making problems, applied in new procedures to determine best compression configuration for the refrigeration cascade.

2. Theoretical Principles of Refrigeration Systems and Heuristics of LNG Plant

In NG liquefaction process, acid gases and mercaptanes are removed from the sour NG. Cascade refrigeration is required to reach very low temperatures. A simplified cascade refrigeration cycle for mega scale LNG plant consists of three sub-cycles, each using a different pure refrigerant, (Figure 1).

Only one stage for each cycle is shown for simplicity while in real industrial cycles 2 or 3 pressure stages operate by using expansion valves where each stage has its own presaturator, economizer, desuperheater,... . In the first cycle, propane leaves the compressor under a high temperature and pressure and enters the condenser where the cooling water or air, is external heat sink. The condensed propane then enters the expansion valve where its pressure is decreased to the evaporator pressure and the temperature of hot streams decreases to -40°C . As the natural gas and methane are cooling down and ethane of lower cycle is being condensed, the liquid refrigerant propane evaporates. Propane leaves the evaporator as superheated vapor and enters the compressor, thus completing the loop. The condensed ethane in the middle cycle expands in the expansion valve and evaporates as methane condenses and then the natural gas is further cooled and liquefied.

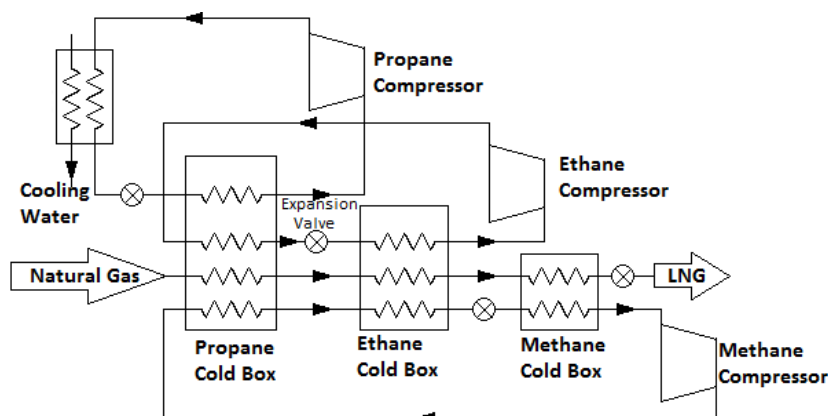


Figure 1. Schematic of cascade refrigeration cycle (Kanoğlu, 2002)

In ethane cycle, temperature of hot streams decreases to -100°C . Finally, the methane expands and then evaporates as natural gas is liquefied and subcooled to -160°C . As methane enters the compressor to complete the loop, the pressure of LNG is dropped to the storage pressure in an expansion valve. Each of these refrigeration cycles have usually three compression and expansion stages and consequently there are three evaporation temperature levels for each refrigerant (Figure 2). Considering the pressure ratios and flow rates in LNG industry, the main compressor type is Centrifugal.

Many refrigeration features are available which can be mounted over simple refrigeration cycles. These options reduce the required compression shaftwork. A cascade refrigeration system and its P-h diagram are shown in Figure 3. Lower cycle absorbs heat at temperature level 1-4 and rejects condensation heat to the upper cycle at temperature level of 2-3. The upper cycle absorbs the rejected heat from the lower cycle by operating at evaporation level of 5-8, which is colder than level 2-3. Finally, the heat in the upper cycle is rejected at level 6-7 to external heat sinks like cooling water and air cooling systems.

The reason for using this kind of cascade refrigeration systems is two-folds. First, there are no single refrigerant in a single cycle to cover all temperature ranges of refrigeration. Figure 4 summarises the operating ranges for some commonly used refrigerants are summarized in (Figure 4) by Lee et al. (2001).

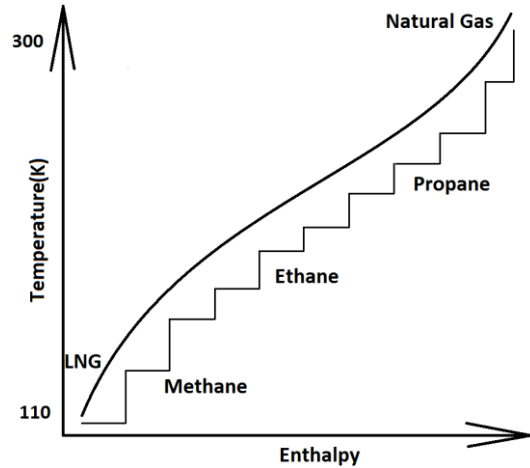


Figure 2. The cooling curve of natural gas and evaporation stages of the refrigerants

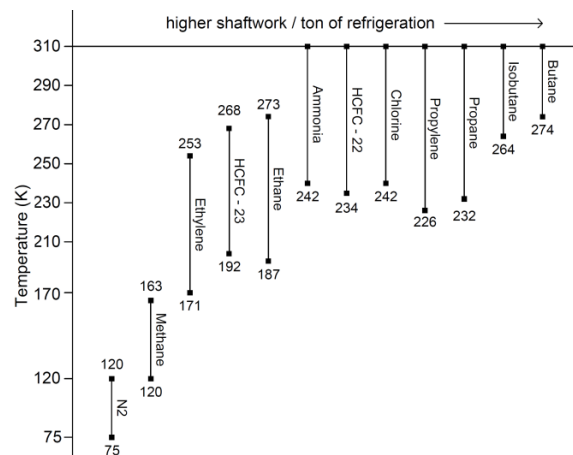


Figure 4. Refrigerant operating range (Lee et al., 2001)

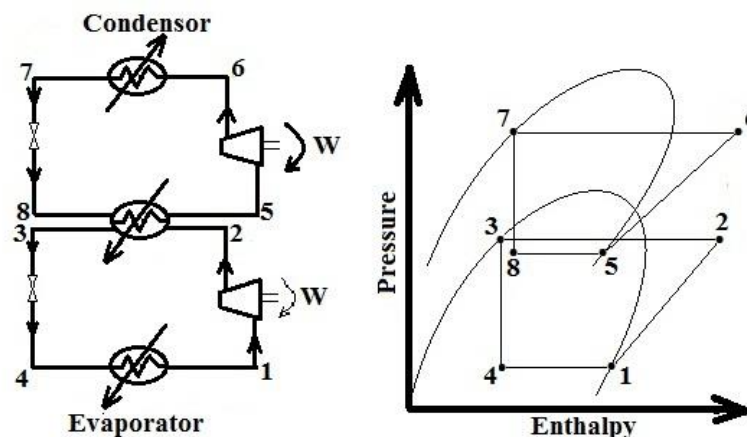


Figure 3. A simple cascade refrigeration system diagram

Second, in terms of energy consumption, using a single refrigerant for the whole refrigeration requirements may consume more shaftwork than using multiple refrigerants. Some basic features of refrigeration in a superstructure are described in section 2.1.

2.1. Refrigeration Features for Shaftwork Saving

For a refrigeration system, it is possible to improve its performance by using the following design options, (Figure 5):

- **Economizer:** Figure 5(i). In an economizer, the condensed refrigerant is flashed into an intermediate pressure level, where the flash vapor is returned to the suction of the compressor and the remaining liquid is further expanded to a lower temperature. As a result, the amount of vapor flowing through the lower pressure part is reduced, thus a saving in the shaftwork.

- **Aftercooler:** Figure 5(i). With this option, the superheated refrigerant vapor is cooled down after compression through other available heat sinks before further compression. This causes a reduction in required shaftwork and the aftercooling duty. Aftercoolers provide the opportunity of heat

integration between refrigeration systems and processes.

- **Presaturator:** Figure 5(iii). A Presaturator has a similar structure as an economizer, but the partially compressed refrigerant vapor is presaturated in the flash vessel with the expense of evaporating some part of the refrigerant liquid from the corresponding economizer. This phenomenon decreases the temperature of the refrigerant vapor entering the next stage of compressor, and saves shaftwork. On the other hand, presaturation may have two drawbacks: (1) it requires a higher refrigerant flow rate which may cause more compression in the shaftwork and (2) both economizer and Presaturator, add an intermediate pressure level, which may cause an increase in capital cost for compressors. Several small compressors can be more expensive than a single big compressor, even though the total shaftwork requirement is reduced.

- **Desuperheater:** Figure 5(iii). Using a Desuperheater, the final stage superheated refrigerant vapor is pre-cooled after compression through a warmer heat sink before entering the condenser. This device adds the possibility of heat integration to processes.

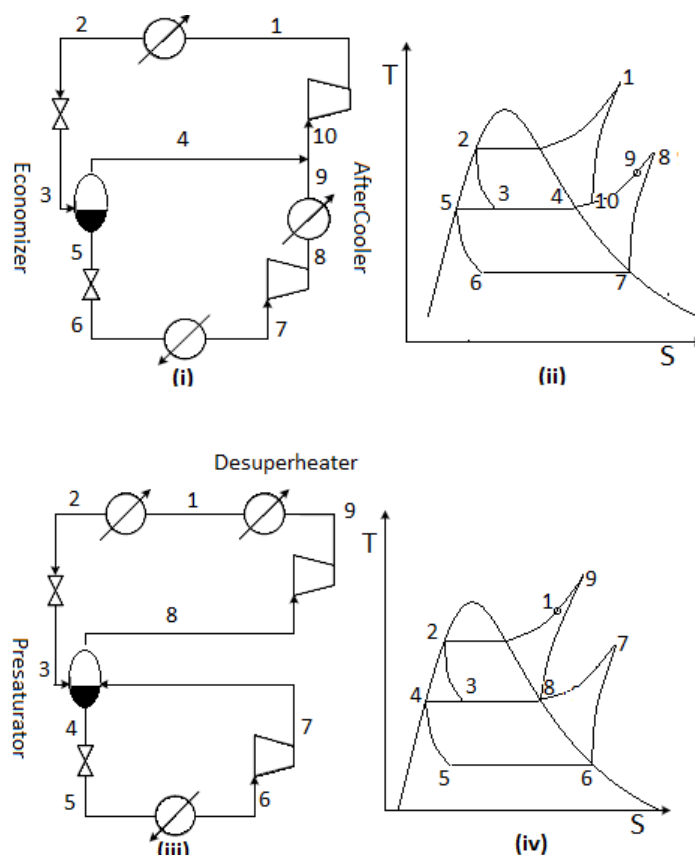


Figure 5. Refrigeration system design options in a cycle

2.2. Determining Compression Configuration Scenarios for Refrigeration

There are many refrigeration configurations. The optimal synthesis of the cascade should be determined for lowest capital and operating cost. The main decision made for a refrigeration cascade include: compression configuration, number of stages in each compression section and refrigeration base temperature that are shown in Figure 6.

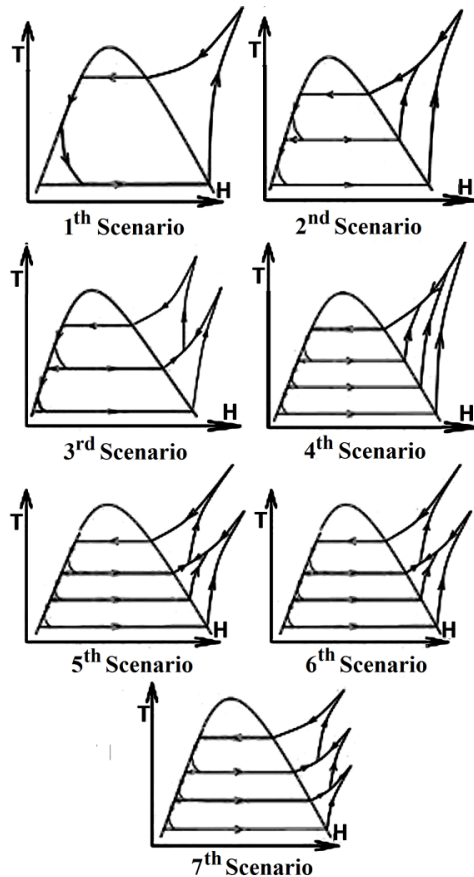


Figure 6. Compression configuration Scenarios

After deciding on these major components, a refrigeration superstructure could be established and then all other refrigeration features could be mounted on the obtained configuration where the superstructure will be optimized.

An important parameter to be determined in a cycle is the number of pressure levels with their associated pressure. Three possibilities are considered for pressure levels in a cycle; any cycle may include one, two, or three pressure levels. The different configuration of compressor, which is a binary variable in our superstructure modeling is presented in Figure 6. Maximum number of sections (volumes which can mechanically increase pressure from inlet

to discharge) in a compressor set is 7 and are placed between pre-specified pressure levels shown in Figure 6.

The above scenarios are mathematically modeled in section 2.4 and MINLP solver finds the best scenario which minimizes cascades shaftwork and capital cost. Different refrigeration features like pre-saturator, economizer, desuperheater shall be mounted over the selected scenario leading to energy consumption reduction.

2.3. Technical Heuristics to Find Best Refrigeration Cascade in LNG Industry

Dealing with complicated problems like multistage refrigeration cascade in some industrial practices with all their constraints pave the way to achieve a realistic and applicable design. The items below are some of the practical guidelines used in cascade system design:

1. The size of LNG plant indicates the complexity of the design. LNG plants with capacities less than 1 Million Ton Per Annual (MTPA) only use one cycle and the designer should avoid a cascade design. This single cycle can be a multistage cycle where all refrigeration features like economizer, presaturator, reboiler, ... are applied. When the LNG plant size increases, it is logical to use two or three cycles in a cascade with same features used in single cycle. (Suprpto, 2007)

2. Bottom temperature of natural gas in a cascade is determined by the required composition of produced LNG. The LNG quality is decided and chosen by main importing market, for example European market requires lower HHV (~970MMBTU/SCF) and East Asia market requires higher HHV (~1100 MMBTU/SCF). When the main market for plant is determined. The specification of the product is known and the lowest required temperature will be found accordingly. This temperature will determine the suction pressure of the lowest cycle compressor.

3. LNG plants are world's largest vapor recompression cycle systems where regarding the pressure ratio and flow rate, the best choice for compressors is the centrifugal type. Compressor manufacturers build compressors which have at most seven stages as in normal practice with a compression ratio of each stage at around 1.7. The pressure levels in the cycle are determined by multiplying the base pressure into 1.7 by the power of the number of sections between corresponding pressure levels (Suprpto, 2007).

4. Each cycle transfers heat load of the process streams and the compressor's work to upper cycle. The recycled refrigerant from higher cycles should be fully condensed, as usually the main heat rejection occurs during condensation. (Gurudath Nayak & Venkatarathnam, 2009)

5. Partition temperatures are the very essential characteristic of any LNG cascade. It divides compression load of refrigeration and temperature range at which cooling occurs. There are two guidelines that help placing the partition temperature between each cycle:

a) Superheated refrigerant that is discharged from lower cycle to the upper cycle should return in liquid phase. (Gurudath Nayak & Venkatarathnam, 2009)

b) Discharge temperature of vapor stream of each compressor not exceeding 135°C (Stoecker, 1998).

By using the developed theoretical principles and these heuristics, the mixed integer non-linear mathematics can model heat-material balance of refrigeration cascade which includes decision making parameters like existence or non-existence of pressure levels and compression configuration and selects among economizer and presaturator while minimizing the capital and operating cost of the plant. MINLP method is used where logical selections or different sets of equations are applied in different design scenario. This method as a decision-making tool, helps to determine the best configuration discussed in section 2.4.

2.4. Mixed Integer Non-Linear Programming Model in Refrigeration Systems

The MINLP is a form to model problems where discrete variables are restricted to values of 0 and 1 and represent certain decisions necessary to deal with continuous variables. (Oldenburg & Marquardt, 2008)

In a refrigeration superstructure a framework is developed by allowing a bypass model to take effect when a given option is eliminated from the superstructure; such a model has the following form:

$$\begin{aligned} \text{Min } Z &= \sum_i \sum_k C_{ik} + d^T y \\ \text{s.t. } &g(x) \leq 0 \\ &r(x) + Dy \leq 0 \\ &Ay \geq a \\ &\begin{bmatrix} Y_{ik} \\ h_{ik}(x, C_{ik}) \leq 0 \\ C_{ik} = \gamma_{ki} \end{bmatrix} \vee \begin{bmatrix} -Y_{ik} \\ B^i x = 0 \\ C_{ik} = 0 \end{bmatrix} \\ &\Omega(Y) = \text{True} \\ &x \in R^n, c \in R^m, y \in \{0,1\}, Y \in \{\text{True}, \text{False}\}^m \end{aligned} \quad (1)$$

Y_{ik} is the Boolean variables that determine whether a given term (heat-material balance) in a disjunction is true [$h_{ik}(x, C_{ik}) \leq 0$] or false [$h_{ik}(x, C_{ik}) \geq 0$], x and C_{ik} are the continuous variables, the latter being used to model annualized costs associated with each disjunction, $\Omega(Y)$ are the logical relations assumed to be in the form of propositional logic involving only the Boolean variables. $g(x) \leq 0$ represents thermodynamic and industrial constraints that are valid over the entire search space while the disjunction $k \in \text{SD}$ states that at least one subset of constraints $h_{ik}(x, C_{ik}) \leq 0, i \in D_{ik}$ must be held (ie, presaturator and an economizer cannot co-exist simultaneously in the same stage), Y_{ik} are the auxiliary variables that control the part of the feasible space where the continuous variable x lie, while the logical condition $\Omega(Y)$, express relations between the disjunctive sets.

The $r(x) + Dy \leq 0$ represents the general mixed-integer algebraic formulations where the original disjunctions are transformed into algebraic equations. $Ay \geq a$ is a set of integer inequalities and d_j^T are linear cost terms. This form is more flexible than rigorous modeling. Disjunctive programming can be used as a basis to formulate a mixed-integer program with 0-1 variables (Oldenburg & Marquardt, 2008).

The fact that, when an option is omitted its related to the constraints or equations which are sufficiently relaxed or unbounded and make the minimization become robust and computationally efficient. Much less time is spent to calculate and converge those virtually non-existing equations.

In each intermediate pressure level of a cycle the existence or non-existence of refrigeration features discussed in section 2.2 and other constraint are modeled through the following mathematical equations.

2.4.1. Vapor-Liquid Heat Exchangers Modeling

In a V-L heat exchanger, the incoming saturated refrigerant liquid exchanges heat with refrigerant vapour after evaporation. The situation when a V-L heat exchanger is placed in the k^{th} level of a refrigeration cycle is shown in Figure 7. Discrete modeling is applied to describe the existence of a V-L heat exchanger (in the k^{th} level:

$$\begin{bmatrix} y_k^V \\ y_k^L + y_{k-1}^L + \dots + y_1^L = 1 \\ T_k^{\text{in}} = T_k^e + \frac{q_k^V}{m \cdot C_p} \\ T_k^{\text{in}} \leq \sum_{i=k-1}^1 y_i^L \cdot T_i^e - \Delta T \end{bmatrix} \vee \begin{bmatrix} y_k^V \\ T_k^{\text{in}} = T_k^e \end{bmatrix} \quad (2)$$

Where y_k^V and $-y_k^V$ denote the existence and non-existence of a vapour heat exchanger at the compressor's suction vapour line, respectively. When a vapour heat exchanger is placed, there must be a liquid heat exchanger so that the energy balance is kept.

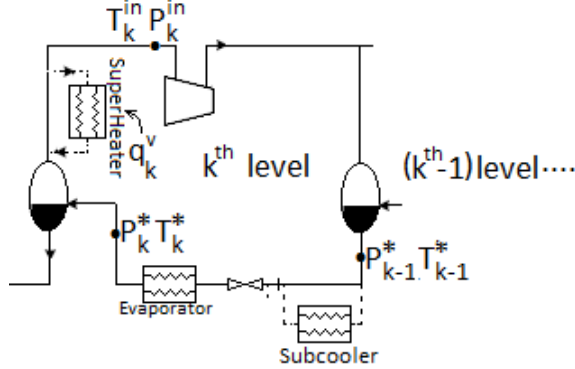


Figure 7. A V-L heat exchanger in the k^{th} level (Lee et al., 2001)

It is possible to have any liquid heat exchanger with saturated temperature higher than or equal to T_{k-1}^e , but only one is needed. This condition is described by the first constraint of the left hand side. Third constraint in equation (3) guarantees validity of heat transfer processes. On the other hand, if vapour heat exchanger is not placed in the superstructure, the condition of compressor's inlet stream is simply set to equal to evaporator's outlet conditions. This discrete model is transformed into mathematical model by using "Big-M" transformation (Oldenburg & Marquardt, 2008):

$$\begin{aligned} (y_k^L + y_{k-1}^L + \dots + y_1^L) + M(1 - y_k^V) &\geq 1 \\ (y_k^L + y_{k-1}^L + \dots + y_1^L) + M(1 - y_k^V) &\leq 1 \\ T_k^{\text{in}} &= T_k^e + \frac{q_k^V}{m \cdot C_p} \cdot y_k^V \\ T_k^{\text{in}} &\leq \sum_{i=k-1}^1 y_i^L \cdot T_i^e - \Delta T + M(1 - y_k^V) \end{aligned} \quad (3)$$

2.4.2. Discrete Modeling of after Cooler

An aftercooler is installed when there is an opportunity of external heat sink, such as air, cooling water or cold utilities. The placement of an aftercooler in the k^{th} level of a refrigeration cycle is shown in Figure 8.

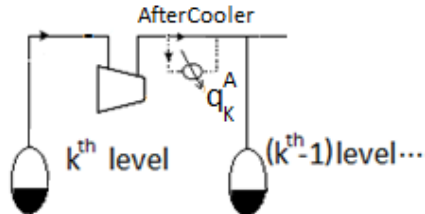


Figure 8. Placement of an aftercooler in the k^{th} level (Lee et al., 2001)

Discrete modeling formulation for installment of an aftercooler is:

$$\begin{cases} y_k^A \\ T_k^{\text{out,A}} = T_k^{\text{out}} - \frac{q_k^A}{m \cdot C_p} \cdot y_k^A \\ T_k^{\text{out,A}} = T_k^{\text{Amb}} + \Delta T \end{cases} \quad (4)$$

Where y_k^A and $-y_k^A$ denote the existence and non-existence of an aftercooler in the k^{th} level. The decrease in temperature, after the aftercooler is calculated by the second constraint. The big-M transformation of this model is:

$$\begin{cases} T_k^{\text{out,A}} = T_k^{\text{out}} - \frac{q_k^A}{m \cdot C_p} \cdot y_k^A \\ T_k^{\text{out,A}} + M(1 - y_k^A) \geq T_k^{\text{Amb}} + \Delta T \end{cases} \quad (5)$$

2.4.3. Discrete Modeling of Presaturator and Economiser

When an economiser is installed, the condensed refrigerant is flashed to an intermediate pressure, where the flash vapour is returned to the suction of the compressor. When a presaturator is installed, the compressed refrigerant vapour is presaturated in the flash vessel with the expense of evaporating part of the refrigerant liquid from the previous level. The dotted line in Figure 9 represents a presaturator option.

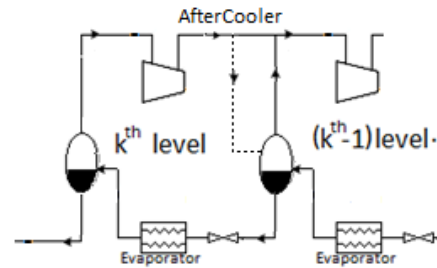


Figure 9. Presaturator option (The dotted line) (Lee et al., 2001)

The reason for discussing the two options together is that they cannot co-exist simultaneously in the same stage. For installing a presaturator, the model is:

$$\begin{cases} y_k^P \\ \dot{m}_{k-1}^{\text{cin,P}} = \dot{m}_{k-1}^{\text{cin}} + \frac{\dot{m}_k^{\text{cout}} \cdot C_p \cdot (T_k^{\text{cout}} - T_{k-1}^e)}{h_{k-1}^{\text{vap}}} \cdot y_k^P \end{cases} \quad (6)$$

And for installing an economizer, the model is:

$$\begin{cases} y_k^C \\ q_k^E = 0 \end{cases} \quad (7)$$

Since a presaturator and an economiser are not possible to co-exist in the same level, the following constraint is required:

$$y_k^P + y_k^C \leq 1 \quad (8)$$

The complete mathematical formulations that describe selection of two options are expressed as:

$$\begin{aligned} m_{k-1}^{cin,P} &= m_{k-1}^{cin} + \frac{m_k^{cout} \cdot C_p \cdot (T_k^{cout} - T_{k-1}^e)}{h_{k-1}^{vap}} \cdot y_k^P \\ q_k^E &\leq M(1 - y_k^C) \\ q_k^E &\geq -M(1 - y_k^C) \\ y_k^P + y_k^C &\leq 1 \end{aligned} \quad (9)$$

The introduced procedure is incorporated in a (LNG-Pro) program which integrates VBA, Refprop and Excel MINLP Solver to automate the methodology. The LNG-Pro begins optimization process from the lowest cycle and after meeting the Heat-Material Balance the MINLP solver determines which integration of the above features minimizes the compression shaftwork. The developed

program starts optimization from lowest cycle and after meeting the Heat-Material Balance (HMB) and above constraints, both the rejected process heat load and the compressor's load are shifted up to highest and middle cycles. The Intermediate pressure and flow rates are constrained not to allow temperature cross in MSHXs. The procedure is accomplished in two steps. First step which determines the main parameter of a cryogenic cascade like partition temperature and pressure level configuration (Figure 10) and step two, where refrigeration features like subcooler, aftercooler, presaturator and economizer are determined (Figure 11).

The procedure is applied on a mega scale LNG plant with the liquefaction capacity of 25 million M³ per day of natural gas that is equal to the upstream production rate of one standard phase of Iran's South Pars Gas field. Design procedure illustrates the method's ease of use, flexibility and applicability.

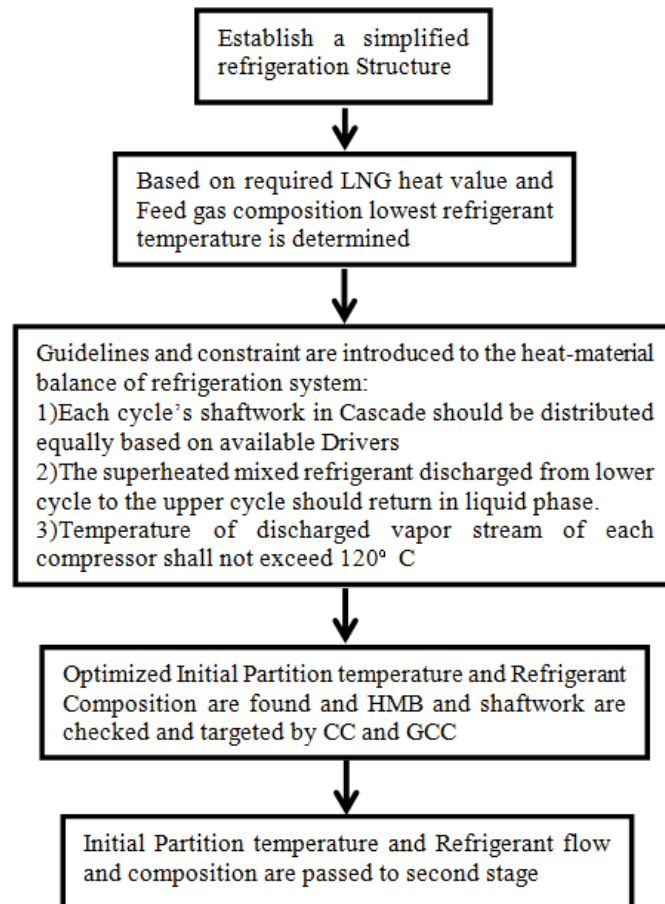


Figure 10. First step of design procedure used by LNG-Pro to find initial partition temperature and refrigerant flow

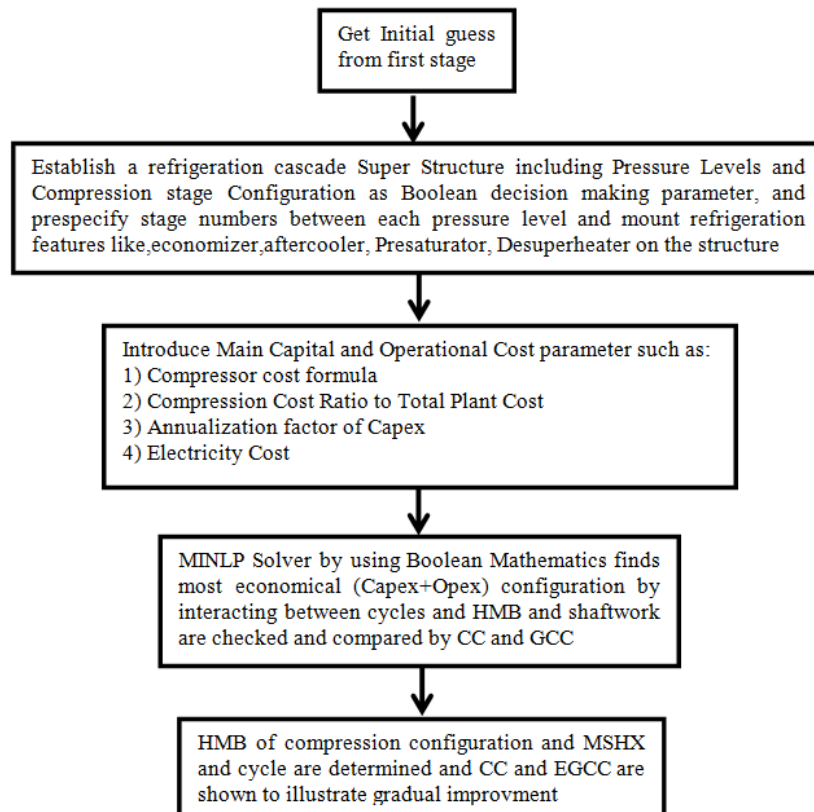


Figure 11. Second step of design procedure used by LNG-Pro to find best compression configuration and refrigeration features by minimizing Capex and Opex

3. Sample description

There is a gas field with a capacity of around 90 BCM and a LNG plant with a 25 year lifecycle. This amount of gas equals to an LNG plant with the capacity of 5.4 MTPA of LNG. Treated natural gas composition is propane 3%, ethane 5%, methane 90%, Nitrogen 2% on mole basis with a pressure of 90 bar. A set of multi stream heat exchanger (MSHX) with approach temperature of 5°C; compressor's with isentropic efficiency of 82% and ambient temperature of 300K is available. Since the plant is sizable, it could be partitioned into three different cycles (Mafi, et al., 2013a, 2013b). The evaporation rate after each expansion valve is assumed to be 5%, 10%, 15% for lowest, middle and highest cycle for initial guess, and the base pressure is 2.25, 1.15, 1.15 bar respectively.

The optimum cascade is modeled in ASPEN PLUS and results are verified by it.

The designing begins by establishing a superstructure by using methods described in section 2 in order to find compression configuration, number of stages in each compression section and the refrigeration base temperature.

Nature of this phase of design is decision making and existence or non-existence of a compression configuration or pressure level introduces 2 different sets of integer variables. Natural gas and heat-material balance equations in the superstructure model are all non-linear. There are thousands of refrigeration configurations that can meet an LNG cycle thermodynamically, but these configurations are constrained by many industrial limits and consider operational and economical parameters to tailor the best multistage cascade cycle for an LNG plant.

In the aforementioned superstructure are annualized all expenses. The electricity cost for compressor driver is assumed to be 0.06 \$ per KWH and compressor's cost as the main single component of a liquefier is estimated to be: $740 * (\text{Shaftwork (KW)}) + 612630$ (Kanoğlu, 2002). The cost of compressor for such a plant is estimated to be around 12% of total plant cost (Peters, Timmerhaus, West, Timmerhaus, & West, 1968).

The LNG-Pro Program begins optimization by applying the aforementioned MINLP formulas in section 2.4. Result summary of this step is represented in table 1.

The Composite Curve (CC) and Grand Composite Curve (GCC) of the selected LNG cascade are shown in Figure 12 and 13. These

curves summarize heat load of hot and cold streams in the refrigeration system. Straight lines indicate evaporation temperature of refrigerants at different pressure levels. The area between hot and cold curves is an indicator of irreversibility and exergy loss in the system. These Curves assure that heat material balance is met and no heat cross between hot and cold stream has taken place.

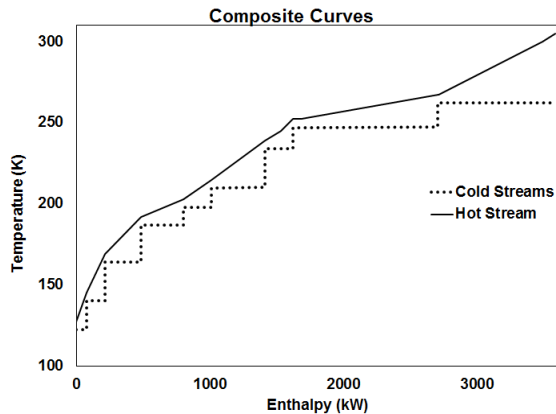


Figure 12. LNG Cycle's Composite Curve after determining main Refrigeration Configuration

Figure 12 shows The pinch point that is the most critical place in this approach is shown in Figure 12. If a more flexible and reliable design is desired, the sharp edges in Composite Curve must be avoided and for saving energy consumption the area between these curves must be decreased.

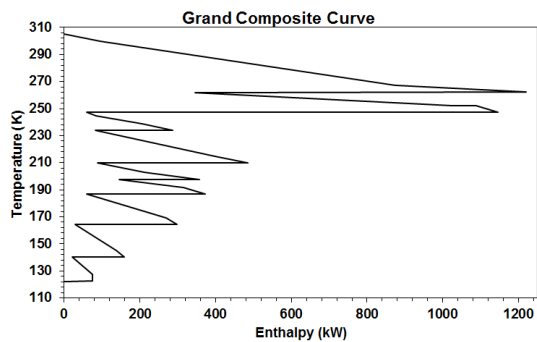


Figure 13.. LNG Cycle's Grand Composite Curve after determining main refrigeration configuration

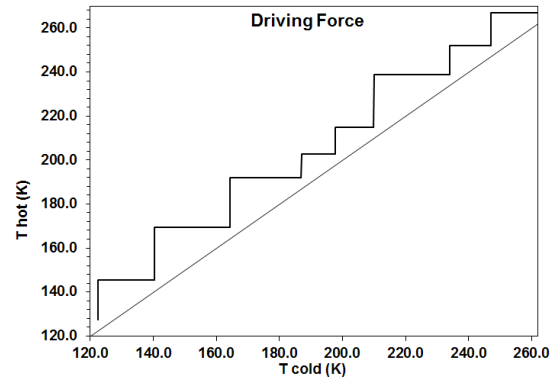


Figure 14. LNG Cycle's Driving Force Curve after determining main refrigeration configuration

If the above assumptions changes, for instance more importance is given to operating cost (Higher electricity price) or capital cost, different configuration will be obtained which are compatible to those conditions. Construction cost for such a plant is anticipated to be 2357.24 million \$.

The base configuration that includes the main parameters of a refrigeration system like pressure levels and where compression configurations are determined is summarized in Table 10. In the next step the complimentary options like, economizer, aftercooler, presaturator and desuperheater, ... are added to this structure

The options in superstructure model are disjuncted by Boolean variables and the problem is modeled through equations 2.4.1 to 2.4.3 and is solved by MINLP Solver Engine in LNG-Pro Program. By applying all aforementioned technics and by considering all operating and capital cost and all possible features the best refrigeration configuration with pure refrigerant is found. The final

Composite Curve, the Grand Composite Curve and Driving force diagrams of the cascade are shown in Figure 15, 17, 19 respectively. The calculated shaftwork for the optimized multistage cascade is 206.87 MW which is well below the first step which was 227.44 MW. This shaft work reduction is a direct result of mounting more refrigeration features, that means more capital cost and a more complicated cycles.

Table 1. Optimal Compression Configuration and refrigerant evaporation pressure level and Minimized Plant's OPEX and CAPEX cost

Cycle Position	Partition temp (K)	Selected Scenario	Pressure levels(bar)	Compressor Section Configuration	Shaftwork (MW)	Annualized Capital Cost(M \$)	Annual Operating Cost(M \$)
High	239	5 th	1.15-1.95-3.32-16.33	1-1-3	127.38	1316.6	63.28
Middle	195	7 th	1.15-1.95-3.32-16.33	1-1-3	62.86	651.1	31.23
Low	122	7 th	2.25-6.5-18.79-54.3	2-2-2	37.2	389.5	18.48

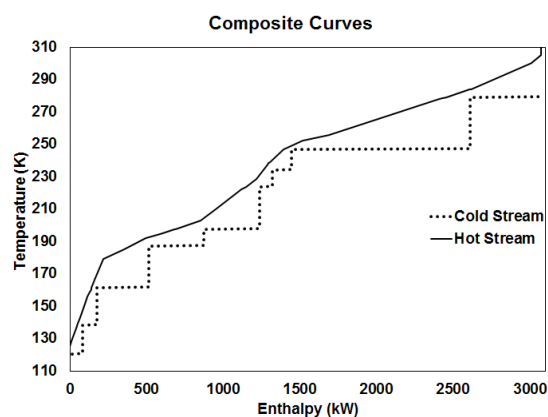


Figure 15. LNG Cycle's Composite Curve including all Refrigeration features

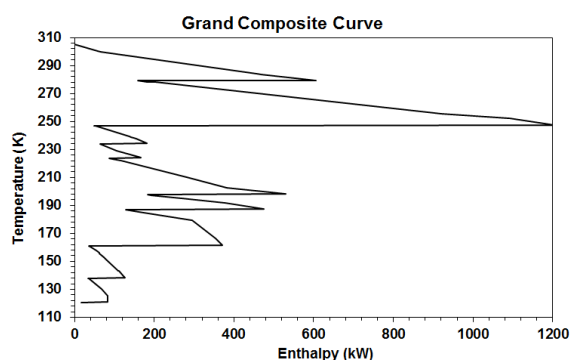


Figure 16. LNG Cycle's Grand Composite Curve including all Refrigeration features

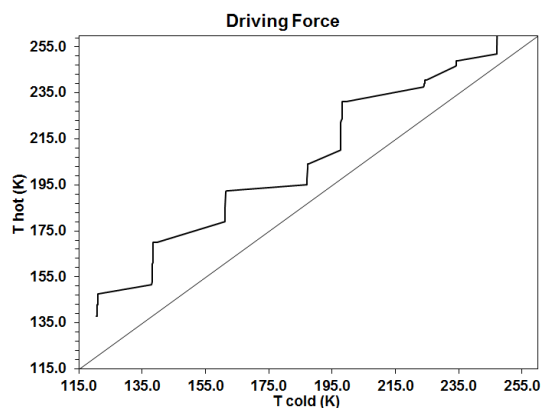


Figure 17. LNG Cycle's Driving Force Curve including all Refrigeration features

Comparison between Composite Curve (CC) and Grand Composite Curve (GCC) of Figure 12 and 13 with Figure 15 and 16 show the step by step progress in design procedure clearly.

Driving force of the preliminary cascade (Figure 15) is reduced in final model's driving force (Figure 17). This reduction results in lower shaftwork consumption of the final cascade.

The heat and power balance for the optimal cascade cycles which is found by this method is summarized in Table 2.

The final model with its features is simulated in ASPEN PLUS and the streamlines are labeled in the schematic cycle. The selected LNG plant needs to have 3 Multistream Heat Exchangers (MSHX) or Cold Box, which correspond to 3 available cycles in the cascade. Each MSHX is divided into three sections because of three available pressure levels in the T-S diagram. The T-S diagrams and relevant HMB tables of it are presented below:

- 1) Lowest cycle's (Coolest Cycle) equipment structure and the T-S Diagram and the relevant Cold Boxes:

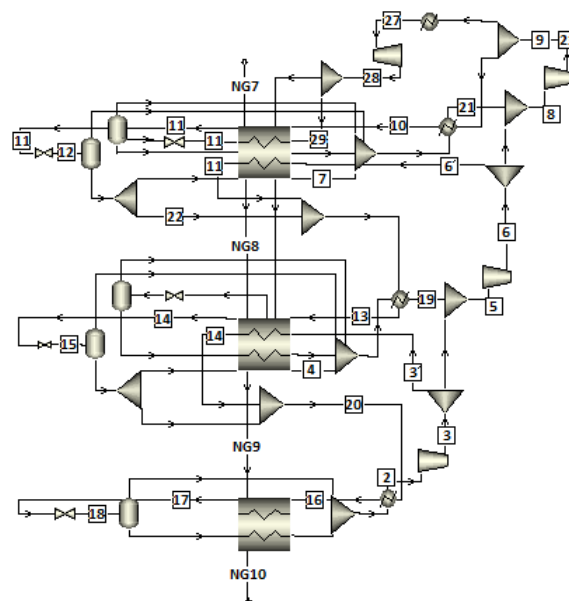


Figure 18. Lower Cycle's equipment structure in Aspen Plus

Table 2. Heat and shaftwork summary of final refrigeration cascade

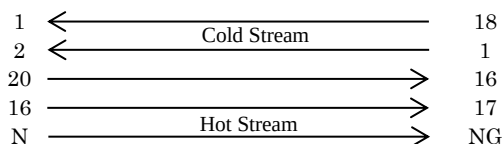
	Lowest Cycle	Middle Cycle	Highest Cycle
NG Cooling rate(KW)	263.96	293.93	209.29
Compressor Shaftwork(MW)	42.54	60.19	104.15
Heat rejected from lower to upper cycle(Kw)	500.88	1092.81	1897.88
Refrigerant Flow rate(kg/s) ratio to NG	Methane	ethane	Propane
	1.5336	2.08	5

Hot and cold streamlines of sub-cooling cycle are packed in a cold box. Heat material

balance for MSHX is represented in Table 4, 5, 6:

Table 4. Low Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	1	1'	2	16	17	18	20	NG9	NG10
Temperature(K)	120.6	120.6	121.1	137.7	133.9	120.6	138.0	143.0	125.6
Enthalpy(KJ/KG)	524.9	524.9	526.1	94.5	80.2	31.6	95.6	121.8	60.3
Flow(KG/S)	0.129	0.143	0.143	0.143	0.143	0.129	0.143	1.000	1.000

**Table 5.** Mid Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	3	3'	4	4'	5	13	14	19	20	22	NG8	NG9
Temperature(K)	159.2	159.2	138.0	138.0	185.8	140.7	140.2	143.0	138.0	161.2	166.2	143.0
Enthalpy(KJ/KG)	613.3	613.3	546.5	546.5	657.7	106.6	104.7	559.1	95.6	190.1	209.3	121.8
Flow(KG/S)	0.143	0.000	0.195	0.202	0.345	0.345	0.345	0.000	0.000	0.345	1.000	1.000

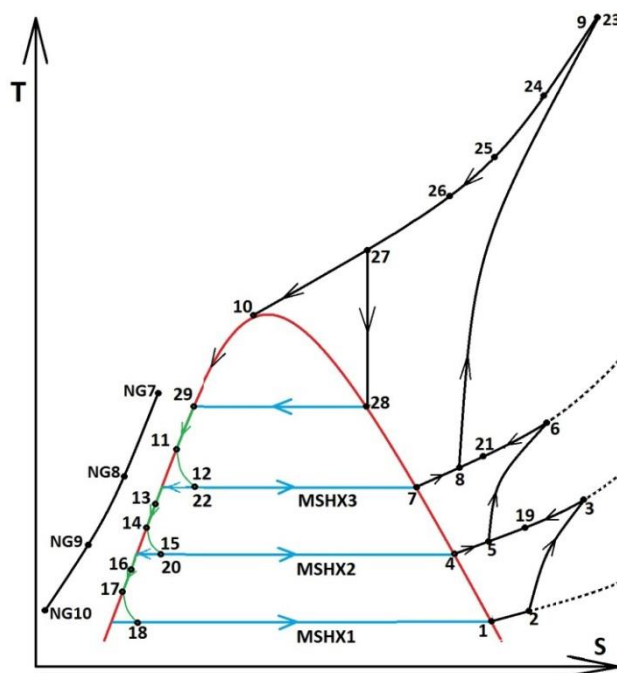
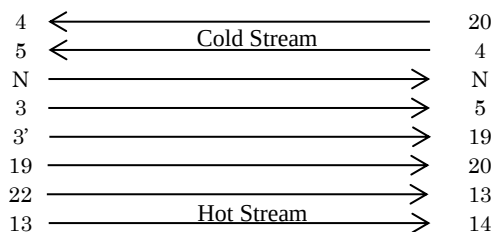
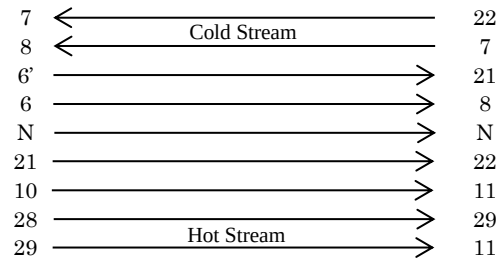


Figure 19. T-S diagram of lowest refrigeration cycle

Table 6. High Pressure Refrigerant Core and the schematic of its hot and cold streamlines

Stream Name	6	6'	7	7'	8	8'	10	11	11'	21	22	22'	28	29	NG7	NG8
Temperature(K)	243.8	243.8	161.2	161.2	161.7	179.3	191.9	180.1	180.1	166.2	161.2	160.7	167.1	164.5	191.9	166.2
Enthalpy(KJ/KG)	793.3	793.3	556.9	556.9	558.6	611.5	399.3	276.3	276.3	573.4	190.1	187.9	591.2	584.0	324.2	209.3
Flow(KG/S)	0.345	0.000	0.828	1.188	1.188	1.534	1.534	1.534	0.000	0.000	0.828	0.000	0.000	0.000	1.000	1.000



2) Middle Cycle's equipment structure and T-S Diagram and its relevant Cold Box:

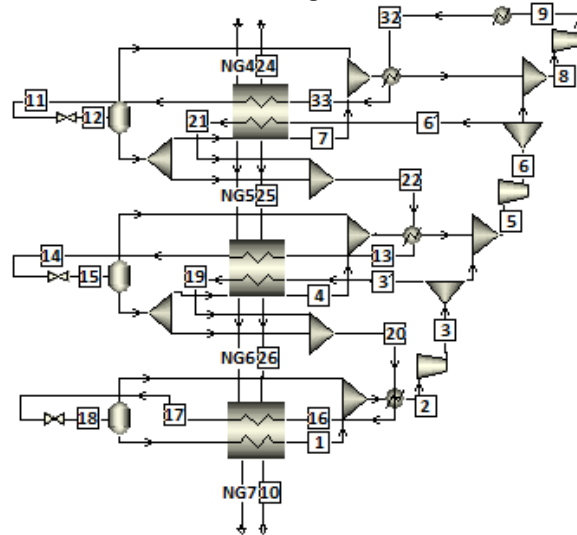
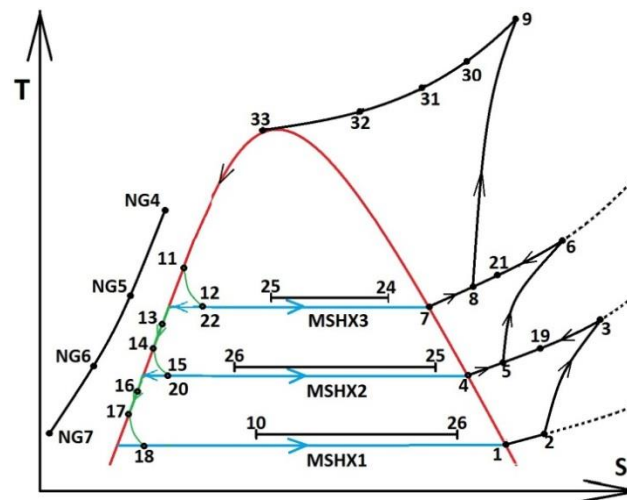
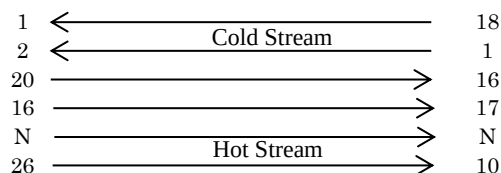
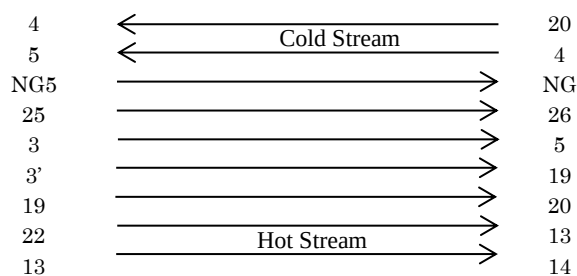
**Figure 20.** Middle Cycle's equipment structure modeled in Aspen Plus**Figure 21.** T-S diagram of Middle refrigeration cycle

Table 7. Low Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

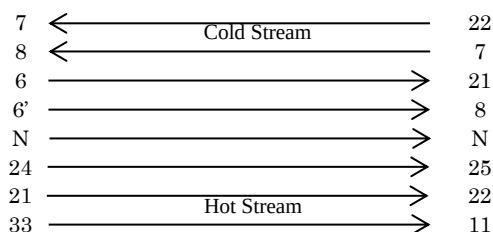
Stream Name	1	1'	2	16	17	18	20	26	10	NG6	NG7
Temperature(K)	186.9	186.9	187.4	197.4	195.2	186.9	197.7	202.7	191.9	202.7	191.9
Enthalpy(KJ/KG)	492.1	492.1	492.8	31.7	26.2	5.8	32.4	570.7	399.3	381.7	321.8
Flow(KG/S)	0.672	0.701	0.701	0.701	0.701	0.672	0.701	1.534	1.534	1.000	1.000

**Table 8.** Mid Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	3	3'	4	4'	5	13	14	19	20	22	NG	NG	25	26
Temperature(K)	2	213.	197.	197.	200.	197.	195.	197.	197.	197.	214.	202.	214.	202.
Enthalpy(KJ/K)	5	526.	503.	503.	506.	31.7	26.2	31.7	32.4	32.4	468.	381.	624.	570.
Flow(KG/S)	0.	0.00	0.36	0.43	0.43	0.70	0.70	0.00	0.00	0.70	1.00	1.00	1.53	1.53

**Table 9.** High Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	6	6'	7	7'	8	8'	33	11	21	22	22	24	25	N	N
Temperature(K)	23	23	20	20	21	22	23	23	22	20	20	23	21	23	21
Enthalpy(KJ/)	55	55	51	51	51	53	14	14	53	63	62	70	62	61	46
Flow(KG/S)	1.	0.	0.	0.	0.	1.	2.	2.	1.	0.	1.	1.	1.	1.	1.



3) Highest Cycle's equipment structure and T-S
Diagram and its relevant Cold Box:

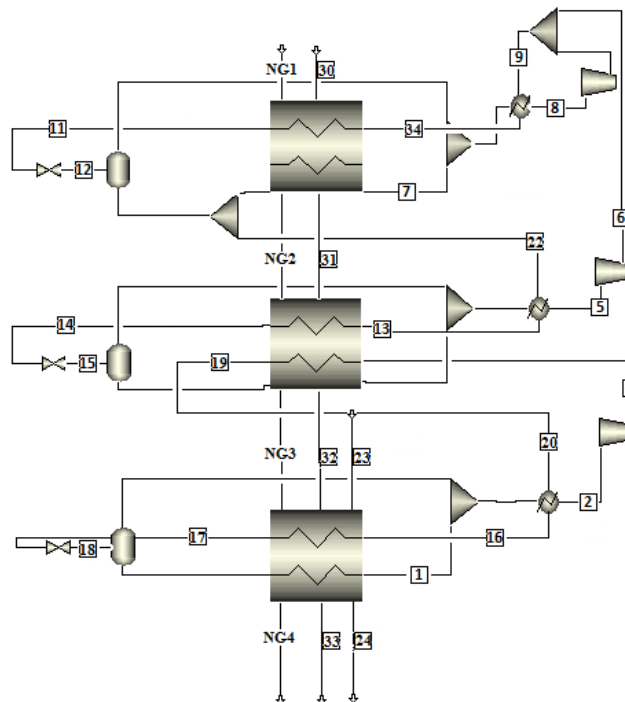


Figure 22. Higher Cycle's equipment structure modeled in Aspen Plus

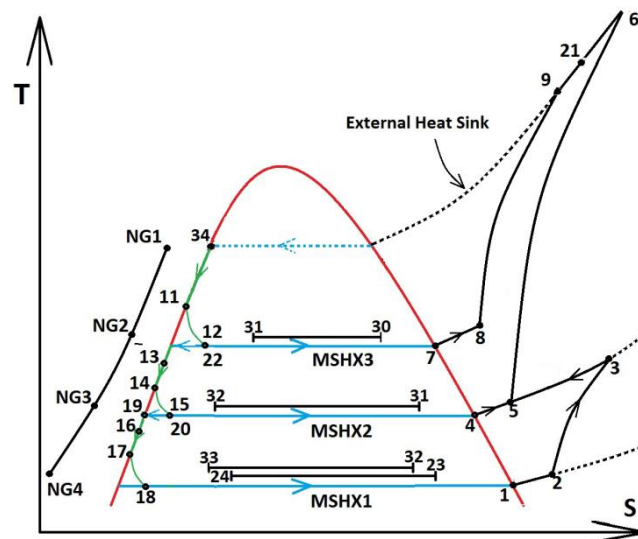


Figure 23. T-S diagram of the Higher refrigeration cycle

Table 10. Low Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	1	1'	2	16	17	18	20	23	24	32	33	NG3	NG4
Temperature(K)	234.0	233.9	234.4	246.8	237.6	233.9	247.1	246.8	239.0	252.1	238.9	252.1	238.9
Enthalpy(KJ/KG)	529.6	529.4	530.2	136.4	115.4	106.9	137.1	725.9	703.6	179.9	141.7	670.3	615.7
Flow(KG/S)	0.419	0.428	0.428	0.428	0.428	0.419	0.428	1.534	1.534	2.084	2.084	1.000	1.000

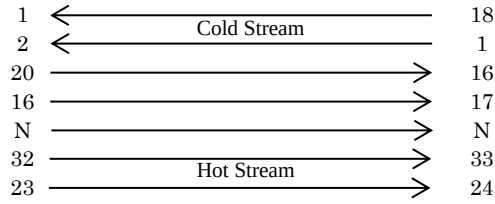


Table 11. Mid Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	3	3'	4	4'	5	13	14	19	20	22	31	32	NG2	NG3
Temperature(K)	255.7	255.7	247.1	247.1	248.0	261.4	260.9	247.1	247.1	262.0	267.0	252.1	267.0	252.1
Enthalpy(KJ/KG)	558.4	558.4	545.0	545.0	546.5	171.1	169.8	137.1	545.0	172.6	568.1	0.0	723.1	670.3
Flow(KG/S)	0.428	0.000	2.564	2.787	2.787	2.787	2.787	0.428	0.428	2.787	2.084	2.084	1.000	1.000

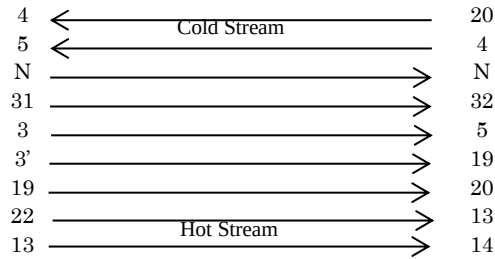


Table 12. High Pressure Refrigerant Core of Cold Box and the schematic of its hot and cold streamlines

Stream Name	6	6'	7	7'	8	8'	34	11	21	22	22'	30	31	NG1	NG2
Temperature(K)	332.5	332.5	262.0	262.0	263.0	263.0	305.0	277.9	262.0	262.0	261.5	300.0	267.0	300.0	267.0
Enthalpy(KJ/KG)	669.5	669.5	562.3	562.3	564.0	564.0	283.9	212.4	172.6	172.6	172.3	666.2	568.1	825.0	723.1
Flow(KG/S)	2.787	0.000	1.702	2.214	2.214	2.787	5.001	5.001	0.000	1.702	0.000	2.084	2.084	1.000	1.000

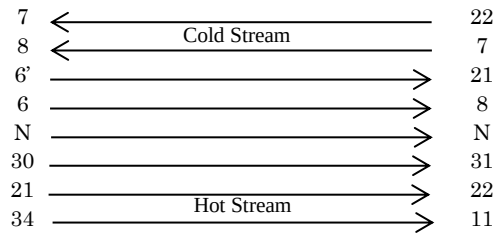


Table 13. Comparison between required shaftwork of LNG-Pro and Aspen-Plus (Kw/Kg)

	Final Model(kw)	Aspen Plus(kw)
First Section of Lower Cycle	12.46	12.54
Second Section of Lower Cycle	46.82	53.41
Third Section of Lower Cycle	175.56	177.79
First Section of Middle Cycle	23.86	25.20
Second Section of Middle Cycle	42.21	48.92
Third Section of Middle Cycle	266.11	268.44
First Section of Higher Cycle	12.10	12.14
Second Section of Higher Cycle	342.73	374.1
Third Section of Higher Cycle	220.06	185.41
Total Compression Work	1141.91	1157.95

Obtained results from LNG-Pro and Aspen-Plus are compared in Table 13 which shows required coincidence for conceptual modeling of introduced procedure.

There is around 1% difference between LNG-Pro and Aspen-Plus in total shaftwork consumption. This deviation arises from some simplifying assumption in LNG-Pro. The main objective of the LNG-pro is to determine the compression configuration and refrigeration features by conceptual procedures and it does not intend to model refrigeration cycles rigorously which is an Aspen-Plus's task. In LNG-Pro, the approach temperature of MSHX is limited to 5°C only at two ends of the exchanger, but in aspen-plus temperature approach is checked through full length of the exchanger. Simplifying assumptions in LNG-Pro is required to speed up the design during decision making stages, which causes around 1% deviation from Aspen-Plus. The main objective of the introduced procedure is to find the minimum annualized cost of an LNG plant by selecting different compression configurations from some pre-defined scenarios. This procedure could find best synthesis for the cascade, shown in Figure 18, 20 and 22.

Pure refrigeration cycles are more energy intensive than the mixed refrigeration cycles but the reliability and operability of these cycles are much higher. Pure refrigeration cycles can operate in off design condition of an LNG plant with higher reliability and efficiency; therefore, if the uncertainty about upstream, market situation and controllability of the plant is a concern, this kind of pure refrigerant cycle is a proper solution.

4. Conclusion

A stepwise design procedure for complex multistage refrigeration system is introduced in this article. The introduced procedure applies a 2 phase disjunctive approach to solve the major decision making parameters in the first phase and deals with the other refrigeration features in the second phase. The important parameters like compression configuration, partition temperature and base pressure are determined in the first phase. At this stage the heuristics which are elaborated in this article to reduce calculation time are used extensively. The MINLP solver tries to satisfy the major refrigeration requirements, like heat material balance equations, equal compressor load sharing for each cycle and rejecting all latent heat of lower cycles superheated refrigerant to the upper cycle. This procedure results in determining best pressure level of each cycle. In

the second phase all refrigeration features like subcooler, economizer, presaturator and after cooler are mounted on top of the found superstructure in the first step and are modeled by developed set of MINLP equations. Then by minimizing the OPEX and CAPEX of the plant, the optimal synthesis of the LNG plant is found. This step is performed by an MINLP solver.

This stepwise procedure is automated in a program (LNG-Pro) and is applied on a 5/4 MTPA LNG plant. The final design has 3 cycles, low, middle and high cycle with a compressor shaftwork of 42.54MW, 60.18MW and 104.15 MW respectively and total required shaftwork of 206.87MW which illustrates a gradual optimization trend in the procedure.

Nomenclature

h^L	Liquid enthalpy
h^V	Vapour enthalpy
H^{VAP}	Latent heat of vaporization
$\dot{m}_k^{Cin}$	The compressor inlet flow rate in the k^{th} level
$\dot{m}_k^{Cin,p}$	The compressor inlet flow rate with the existence of a presaturator in the k^{th} level
$\dot{m}_k^{Cout}$	The compressor outlet flow rate in the k^{th} level
$\dot{m}_k^R$	The refrigerant flow rate in a reboiler of the k^{th} level
M	An arbitrary large number used in Big-M transformation
p_k^e	Evaporation pressure of the k^{th} level
p_k^{in}	Inlet pressure of the k^{th} compressor.
$p_k^{out,A}$	The outlet pressure of an aftercooler in the k^{th} level
q_k^A	The amount of heat transferred into an aftercooler in the k^{th} level
q_k^R	The amount of heat transferred into a reboiler in the k^{th} level
q_k^V	The amount of heat transferred into a V heat exchanger in the k^{th} level
$T_k^{out,A}$	The outlet temperature of an aftercooler in the k^{th} level
T_k^e	Evaporation temperature of the k^{th} level
y_k^A	A binary variable denotes the existence of an aftercooler in the k^{th} level
y_k^C	A binary variable denotes the existence of an economiser in the k^{th} level
y_k^P	A binary variable denotes the existence of a presaturator in the k^{th} level
y_k^V	A binary variable denotes the existence of a V heat exchanger in the k^{th} level

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