

Predictive Control of Gas Injection in Natural Gas Transport Networks

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Abstract: The present sought to draw a comparison between Model Predictive Control performance and two other controllers named Simple PI and Selective PI in controlling large-scale natural gas transport networks. A nonlinear dynamic model of representative gas pipeline was derived from pipeline governing rules and simulated in SIMULINK® environment of MATLAB®. Control schemes were designed to provide a suitable pressure at consumers' nodes by varying the refinery production rate and compressor station output pressure in the middle of pipeline. The results showed that the model predictive control significantly outperformed the other two methods in economic controlling of pipeline by using less energy in compressor station and simultaneously rejecting disturbances. Although MPC controller performed better, it had a more complicated structure and difficult design procedure than PI controllers.

Keywords: Gas Transport Pipeline, Model Predictive Control, PI Control, Selective Control

1. Introduction

Gas transport pipelines is one of the most important methods in transporting gas from gas wells to consumers. Since gas pressure is dropping in long distances, compressor stations should be installed in appropriate places. In view of consumption variation of consumers, the producer should inject an appropriate amount of gas in pipeline and control the output pressure of compressor stations. Gas injection flow and compressor stations' output pressures are two important parameters in gas pipeline operation. For better operation, the consumption rate of all nodes, pipeline dynamics and delay times should be identified. Current pipeline control involves an offline manual control by considering supply and demand all over the gas network. The main objective of researchers is to design a control scheme capable of minimizing the energy

consumption while keeping the system in the safe region. The safe region conditions are involved in are critical points such as minimum delivery pressure, minimum and maximum compressor flow rate. (Marques & Morari, 1988).

The first step in designing a model based control scheme is the modeling step. After that, different control schemes could be surveyed to choose the best one. The mathematical model of an unsteady state gas flow includes several partial differential equations that depend on spatial coordinates and time (Ke & Ti, 2000). An isothermal gas flow was modeled without ignoring any terms in momentum equation (Osiadacz, 1987). Additionally, an unsteady state gas flow in pipeline was modeled assuming a steady state heat transfer term and the constant compressibility factor (Tentis et al., 2003). Their work showed a vital difference

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between pressure profiles of isothermal and non-isothermal models. In another work in 2009, Gonzales et al. modeled an unsteady state flow in pipeline network with the help of MATLAB®/SIMULINK® software (Herran-Gonzales et al., 2009). They have developed two simplified models and solved equations by using Crank-Nicolson algorithm and the method of characteristics.

The Energy Information Administration of USA reported that nearly half of the natural gas price for residential customers came from transportation costs (EIA, 2007). This created a great motive for researchers for reducing high operational cost of gas transport pipelines. One of the early attempts to develop a rational control policy is represented by (Batey et al., 1961), in which some rules of thumbs are presented for operating the system with low energy consumption. In 1988, Marqués and Morari presented a moving horizon optimization formulation, in which a dynamic simulator computed optimal operating profiles for a single source pipeline network (Marques & Morari, 1988). Moreover, compressor performance was optimized by a dynamic optimization formulation in order to follow a desired line pack profile (Abbaspour et al., 2007). Since 2001, predictive control has been used in control and optimization of pipeline operation. In this year, a linear predictive control for industrial oxygen distribution pipeline networks was developed (Zhu et al., 2001). In a recent study, a nonlinear predictive control was used for the operation costs of the optimization

of transport pipeline network (Gopalakrishnan & Biegler, 2013). In contrast with other set point tracking predictive controllers, their controller had an operation cost function and tried to reduce it. They reported that an economic predictive control could greatly reduce the operational costs of pipeline.

In this article, the pipeline network was over 300 Km long from Khangiran refinery to Farooj compressor station in Iran. This pipeline network, after necessary simplifications, includes 11 consumers, one compressor station in the middle of the line and one refinery at the beginning of the line as a producer. The authors developed a precise dynamic gas pipeline network model based on continuity, momentum and energy balances. Afterwards, three control schemes were designed for this pipeline and their performances were evaluated.

2. Dynamic Modeling of Gas Pipeline Networks

Figure 1 is a schematic of gas pipeline considered in this paper. In this figure nodes 1 and 8 represent refinery and compressor station, respectively. Other nodes are consumers. The pipeline network is divided into two sections: section one consists of node 2 to 7 and their pipelines. The rest of the pipeline network, after node 8, is called section two. The diameter, length and elevation change of each pipe segment is shown in Table 1. The conditions of the inlet gas and the flow rate of each consumer are shown in Table 2.

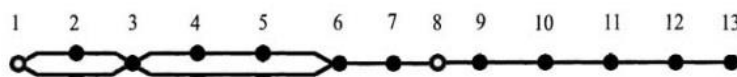


Figure 1. Pipeline Network

Table 1. Pipe Segments Characteristics

Pipe Segment	Outside Diameter (in)	Pipe Thickness (in)	Length (km)	Elevation Change (m)
1-2	36	0.406	49.5	627
2-3	36	0.406	21.5	224
3-4	36	0.406	68.8	182
4-5	36	0.406	10	13
5-6	36	0.406	7	15
6-7	30	0.375	6	19
7-8	30	0.375	19	92
8-9	30	0.375	15.6	89
9-10	30	0.375	42.5	-31
10-11	30	0.375	20.5	101
11-12	30	0.375	35.7	88
12-13	30	0.375	24.5	-21

Table 2. Inlet and Outlet Conditions of Pipeline Network

Pipeline injection pressure (bar)	68.3
Pipeline injection temperature (°C)	38
Consumption flow of node 2 (MMSCMD)	0.014
Consumption flow of node 3 (MMSCMD)	7.200
Consumption flow of node 4 (MMSCMD)	2.768
Consumption flow of node 5 (MMSCMD)	8.953
Consumption flow of node 6 (MMSCMD)	7.790
Consumption flow of node 7 (MMSCMD)	2.886
Consumption flow of node 9 (MMSCMD)	0.395
Consumption flow of node 10 (MMSCMD)	0.504
Consumption flow of node 11 (MMSCMD)	0.019
Consumption flow of node 12 (MMSCMD)	0.603
Outlet flow of pipeline, node 13 (MMSCMD)	10.589

2.1. Pipeline Mathematical Model

The main equations of the dynamic model of gas pipeline are equations of mass, momentum and energy balances, which are expressed as follows:

Mass balance equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (1)$$

Momentum balance equation

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + P)}{\partial x} + \frac{f \rho u |u|}{2d_i} + \rho g \sin(\alpha) = 0 \quad (2)$$

Energy balance equation of gas

$$\frac{\partial}{\partial t} \left[\left(\frac{1}{2} u^2 + U \right) \rho \right] + \frac{\partial}{\partial x} \left[\left(\frac{1}{2} u^2 + H \right) \rho u \right] + \rho g u \sin(\alpha) + h_i \frac{4}{d_i} (T - T_w) = 0 \quad (3)$$

Energy balance equation of pipe wall

$$c_{pw} \rho_w \frac{dT_w}{dt} = [h_i d_i (T - T_w) - h_o d_o (T_w - T_s)] \left(\frac{4}{d_o^2 - d_i^2} \right) \quad (4)$$

The main assumptions in the above model are a) neglecting the radial changes of variables (radially lumped and axially differential), b) neglecting the conduction terms in momentum and energy equations and c) constant properties of pipe wall (c_{pw} and ρ_w). By solving equations 1 to 4 simultaneously, one can achieve velocity (flow rate), pressure and temperature profiles along the pipeline according to time. Required initial conditions are steady state values of temperature, pressure, and flow rate along the pipeline which are calculated by steady state assumptions in the model equations. Boundary conditions are pressure, flow rate, and temperature changes by time in the boundaries

of pipeline (inlet, outlet and consumers). Each pipeline has one inlet and one outlet that needs three boundary conditions on pressure, flow rate, and temperature. For each consumer, one more boundary condition is required. These boundary conditions can be defined at inlet or outlet. In our simulation, pressure and temperature are defined at inlet, and flow rate is defined at outlet and consumers (see Table 2). Compressor energy consumption is formulated as

$$BHP = \frac{H_i m}{550 \eta_c} + 0.75 \left(\frac{H_i m}{550 \eta_c} \right)^{0.4} \quad (5)$$

Where BHP is isentropic power (hp), H_i is isentropic head (lb_f ft/lb_m), m is mass flow rate (lb_m/s) and η_c is isentropic efficiency. H_i is calculated continuously with aim of operating conditions and characteristic curve of compressors. The more BHP means the more energy consumed in compressor stations for compressing natural gas. By recording the BHP of compressor stations with time, one could find which control scheme used less energy to deliver natural gas to consumers and reject disturbances.

2.2. Model Numerical Solution

In order to solve the equations 1 to 3 it was necessary to use numerical solution of partial differential equations methods. To solve equations with method of line (Pregla & Pascher, 1989), they were formed as a set of ordinary differential equations (Kumar, 1987; Chaczykowski, 2010):

$$\begin{aligned} \frac{dP_j}{dt} = & \left(\frac{z \bar{R}}{c_p - Jz \bar{R}} \right) \left\{ \left(\frac{Jz \bar{R} m_j T_j}{A P_j} \right) \Delta_x(P_j) - \left(\frac{m_j c_p}{A} \right) \Delta_x(T_j) - \right. \\ & \left. \left(\frac{Jz \bar{R} T_j}{A} \right) \Delta_x(m_j) - \frac{4h_i(T_j - T_{wj})}{d_i} \right\} - \left(\frac{z \bar{R} T_j}{A} \right) \Delta_x(m_j) \text{ for } j = \\ & 2 \text{ to } N+1 \end{aligned} \quad (6)$$

$$\frac{dm_{j-1}}{dt} = \left(\frac{z\bar{R}m_{j-1}^2 T_{j-1}}{P_{j-1}^2 A} - 144A g_c \right) \Delta_x(P_j) - \left(\frac{2z\bar{R}m_{j-1} T_{j-1}}{AP_{j-1}} \right) \Delta_x(m_j) - \left(\frac{z\bar{R}m_{j-1}^2}{AP_{j-1}} \right) \Delta_x(T_j) - \frac{g \sin(\alpha) P_{j-1}}{z\bar{R}T_{j-1}} - \frac{f z\bar{R}T_{j-1} m_{j-1} |m_{j-1}|}{2Ad_i P_{j-1}} \quad \text{for } j = 2 \text{ to } N+1 \quad (7)$$

$$\frac{dT_j}{dt} = \left(\frac{z\bar{R}T_j}{c_p P_j - Jz\bar{R}P_j} \right) \left\{ \left(\frac{Jz\bar{R}m_j T_j}{AP_j} \right) \Delta_x(P_j) - \left(\frac{m_j c_p}{A} \right) \Delta_x(T_j) - \left(\frac{Jz\bar{R}T_j}{A} \right) \Delta_x(m_j) - \right\} \quad \text{for } j = 2 \text{ to } N+1 \quad (8)$$

where j is the spatial coordinate discretization section index, and P_j , T_j and m_j are pressure (psia), temperature (R) and mass flow rate (lb_m/s) of gas at j -th discretized section of pipe,

respectively. $\Delta_x(P_j)$, $\Delta_x(m_j)$ and $\Delta_x(T_j)$ are approximate differentiation formulas based on backward difference method. A step size of 1640 ft (0.5 km) is used for x -coordinate in these approximate differentiation formula. The value of parameters involved in the model equations (Eq. 4-8) or the method used for calculating these parameters are shown in Table 3. The model equations are solved in MATLAB/SIMULINK using integration routine ODE15s (Natick, 1997).

Table 4 shows a comparison between actual and simulated node's pressure at steady state condition. As seen, most of the node's simulated pressures are in accordance with actual pressures.

Table 3. Model Parameters and Coefficients

Parameter	Description	Formula or Value	Reference
J	Energy conversion factor Btu / (psia ft ³)	0.185	--
g_c	Force conversion factor lb _m ft / (lb _f s ²)	32.174	--
g	Acceleration of gravity (ft/s)	32.174	--
$\bar{R}$	Mass universal gas constant psia ft ³ / (lb _m R)	0.669	--
c_p	Specific heat of gas Btu / (lb _m R)	Based on the method described in chapter 3	(Kumar, 1987)
z	Compressibility factor of gas	Standing and Katz's chart	(Kumar, 1987)
A	Inside cross sectional area of pipe (ft ²)	$\pi d_i^2 / 4$	--
f	Moody friction factor	$\left[1.14 - 2 \log \left(\frac{\varepsilon}{d_i} + 21.25 Re^{-0.9} \right) \right]^{-2}$	(Swamee & Jain, 1976)
h_i	Inside heat transfer coefficient, Btu / (s ft ² R)	$\frac{k_g}{d_i} (0.023 Re^{0.8} Pr^{0.4})$	(Perry & Green, 1997)
h_o	Outside heat transfer coefficient, Btu / (s ft ² R)	$1/(R_{pipe} + R_{soil})$ where $R_{pipe} = d_o \ln \left(\frac{d_o}{d_i} \right) / (2 k_w)$ $R_{soil} = d_o \ln \left(\frac{2d_s + (4d_s^2 - d_o^2)^{0.5}}{d_o} \right) / (2 k_s)$	(Mokhatab et al., 2006)
d_i	Inside diameter of pipe, ft	2.932 or 2.437	--
d_o	Outside diameter of pipe, ft	3 or 2.5	--
d_s	Depth from top of soil to pipe centerline, ft	4.781 or 4.531	--
k_s	Thermal conductivity of soil, Btu / (s ft R)	8.35×10^{-5}	(Perry & Green, 1997), Table 2-382
k_w	Thermal conductivity of pipe (mild steel), Btu / (s ft R)	7.22×10^{-3}	--
c_{pw}	Heat capacity of pipe (mild steel), Btu / (lb _m R)	0.12	(Perry & Green, 1997), Table 2-219
ρ_w	Mass density of pipe (mild steel), lb _m / ft ³	487	(Perry & Green, 1997), Table 2-118
k_g	Thermal conductivity of gas Btu / (s ft R)	4.77×10^{-6}	--
T_s	Temperature of soil (R)	519	--

Table 4. Comparison between Actual and Simulated Node's Pressure

Nodes	Simulated pressure (bar)	Actual Pressure (bar)	Relative Error (%)
2	60.14	61.07	1.52
3	56.66	57.15	0.85
4	55.27	56.1	1.47
5	55.04	55.41	0.66
6	48.21	48.44	0.47
7	47.35	45.62	-3.79
9	58.61	60.03	2.36
10	55.21	57.32	3.68
11	50.2	52.87	5.05
12	45.38	45.87	1.06

3. Control Scheme Design

Pipeline pressure control is required for supplying the consumers' gas needs. In this article, three control schemes, model predictive control, simple PI, and selective PI are surveyed. The control schemes regulate pipeline pressures near their set points for better consumers' supply. All three schemes try to control pressure and supply consumers in an economic way without violating physical constraints of production and distribution tools.

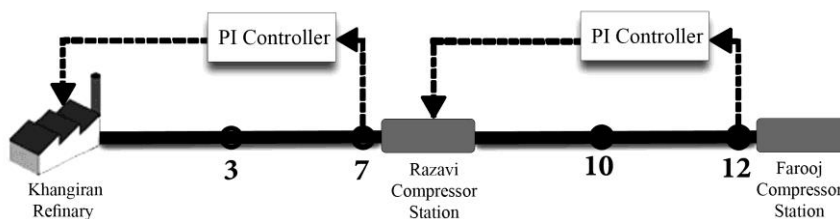
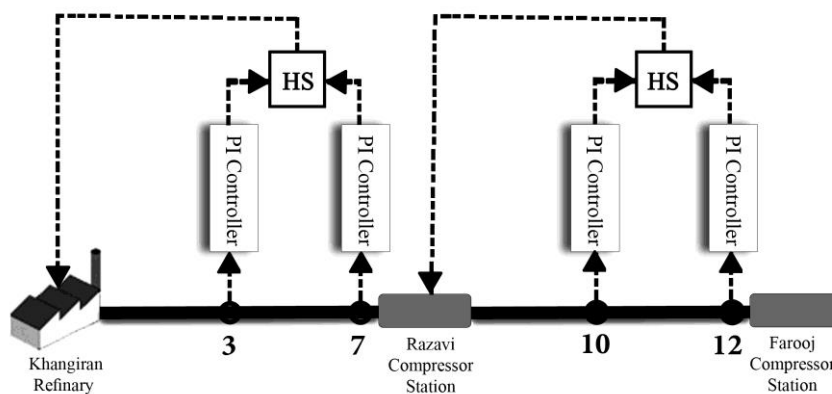
3.1 Simple PI Controller Scheme

In this scheme, two simple PI controllers were used to regulate pressures of nodes 7 and 12 at the end of each section of pipeline. Changing

the consumption rate of a node will result in the pressure change of that node. This pressure change will be sensed in the controlled node with a dynamic that depends on the node distance to the end of pipe section. The controller regulates output pressure of compressor station or refinery to reject this disturbance. This scheme showed in Figure 2.

3.2. Selective PI Controller Scheme

There are only two PI controllers in the simple PI scheme for pressure control. In that scheme, if a disturbance occurs in the middle of pipeline, its effects reach the controlled node by a long dynamic and the controller responds with a huge delay accordingly. The other problem of that scheme is that if the consumption rate of an important node in the middle of the pipeline is disturbed, its pressure will change dramatically before the controller can respond. These reasons establish a motive to put some controllers in the middle of the line to avoid these problems. In the selective controller scheme, nodes 3 and 7 assign refinery injection pressure and nodes 10 and 12 assign the output pressure of compressor station. It should be noticed that consumers in the second section of pipeline indirectly control the refinery injection pressure. When a disturbance occurs in the second section, the controller regulates the output pressure of compressor station which leads to a change in the inlet pressure of compressor station. This change measured is in node 7 and following that, the controller changes the refinery injection pressure. This scheme showed in Figure 3.

**Figure 2.** Simple PI Controller Scheme**Figure 3.** Selective PI Controller Scheme. HS: High Selector

In simple and selective PI schemes SIMC (Skogestad, 2003) method was used for PI controllers design. By considering PI controller in equation 9, controller parameters are shown in Table 5. It should be noticed that controllers for node 7 and 12 are common between simple and selective PI schemes.

$$C(s) = K_c + K_I \frac{1}{s} \quad (9)$$

Table 5. PI Controller Parameters

Nodes	K_c (%/%)	K_I ((%/ %)/s)
3	0.9087	0.0003368
7	0.4769	0.0001135
10	0.9722	0.0006093
12	0.6701	0.0001809

3.3. Model Predictive Control Scheme

Model predictive control (MPC) refers to a class of controllers that compute a manipulated variable profile with the help of a process model to optimize controller performance and avoid constraints violation over a future time horizon (Muske & Rawlings, 1993).

The main idea of model predictive controllers can be summarized as

- Predict the future behavior of the process over the finite time horizon
- Compute the future input signals on line at each step by minimizing a cost function under equal and unequal constraints on the manipulated and controlled variables
- Apply on the plant only the first element of the control variable vector and repeat the previous step with new measured output variables (Zheng, 2011)

In MPC, the vector of control variable is obtained by solving an optimization problem at current time k (MATLAB®, 2010):

$$\begin{aligned} & \text{Minimize } J \\ & = \sum_{i=0}^{p-1} \left(\sum_{j=1}^{n_y} \left| w_{i+1,j}^y (y_j(k+i+1|k) - r_j(k+i+1)) \right|^2 \right. \\ & \quad + \sum_{j=1}^{n_u} \left| w_{i,j}^{\Delta u} \Delta u_j(k+i|k) \right|^2 \\ & \quad \left. + \sum_{j=1}^{n_u} \left| w_{i,j}^{\Delta u} (u_j(k+i|k) - u_{jtarget}(k+i)) \right|^2 \right) \end{aligned} \quad (10)$$

w.r.t

$\Delta u_j(k+i|k)$

s.t

process model (Eqs. 4-8) is satisfied

55.15 bar < refinery injection pressure < 72.4 bar

55.15 bar < compression station outlet pressure < 72.4 bar

refinery injection flow rate < 45.32 MMSCMD

Where subscript j denotes the j -th component of a vector, $(k+i|k)$ denotes the value predicted for time $k+i$ based on the information available at time k and $r(k)$ is the current sampled value of setpoint. $w_{i+1,j}^y$ and $w_{i,j}^{\Delta u}$ are nonnegative weights for the corresponding variable. The smaller w , the less important is the behavior of the corresponding variable to the overall performance. In our work, the MPC toolbox of MATLAB® (R2011a) was used.

MPC is a centralized controller in which controlled variables, set points, and flow rates of four nodes are inputs and manipulated variables are outputs. MPC controller calculates manipulated variables, which are refinery injection pressure and compressor station output pressure, to reject the node's pressure disturbances. The MPC scheme is shown in Figure 4 and the value of its parameters are presented in Table 6.

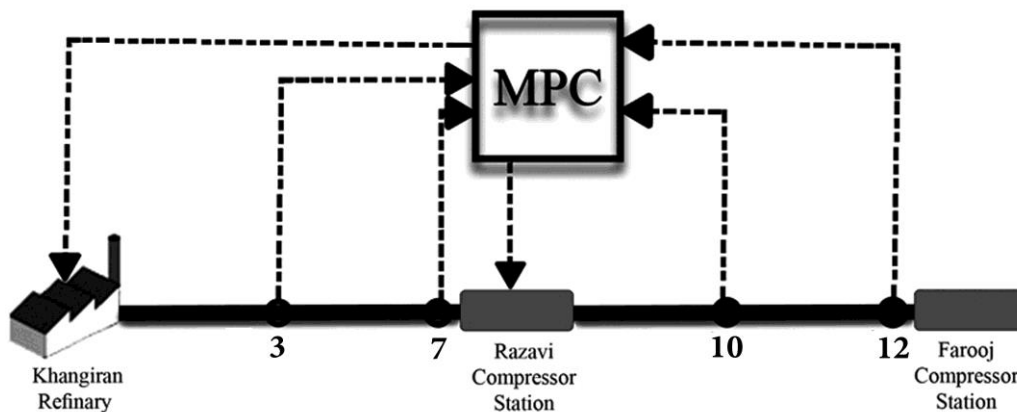


Figure 4. MPC Controller Scheme

Table 6. MPC Designed Parameters

Control Interval (Sampling time)	50 s
Prediction Horizon	600 sampling number
Control Horizon	5 sampling number
Node 3 weight	1
Node 7 weight	2.5
Node 10 weight	1
Node 12 weight	2.5

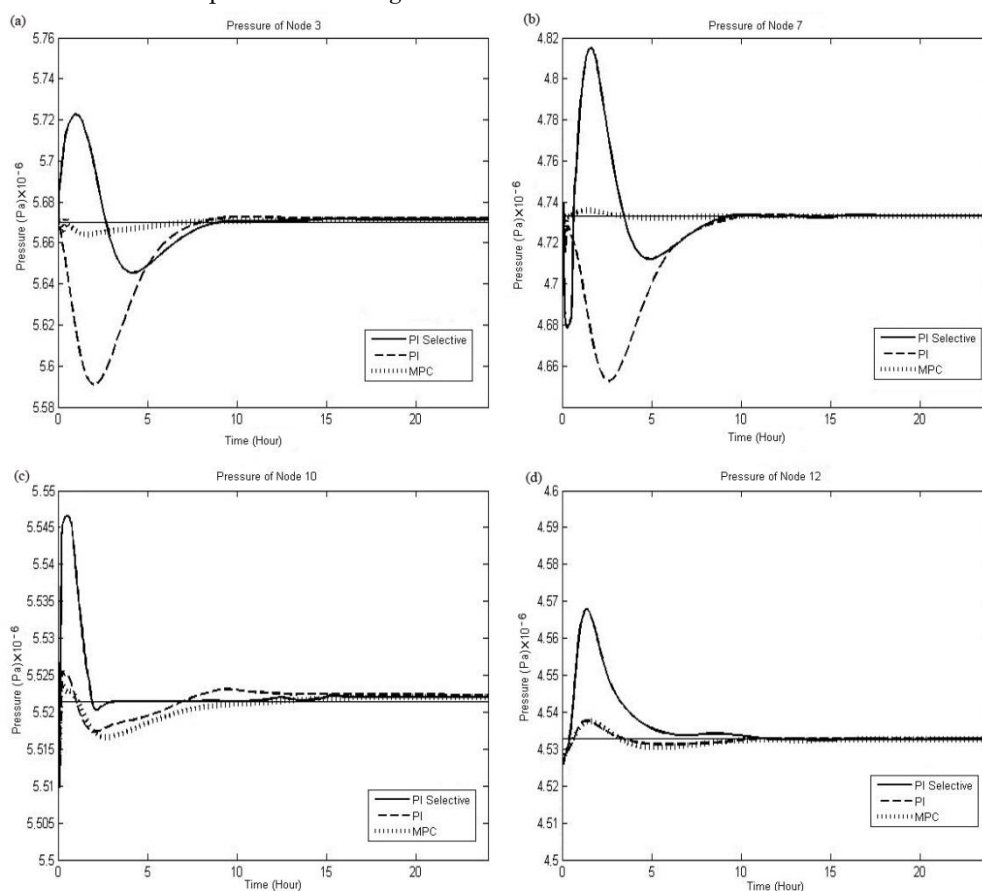
The MPC toolbox in MATLAB software supports linear time invariant model formats such as transfer function and state space models. These models can be imported by user or could be calculated by MATLAB linearization function based on the simulated nonlinear model. In the current work, the second method was used.

4. Results and Discussion

Three designed control schemes were applied to the simulated gas pipeline network. To compare the controller's performance, three experiments were performed. In these experiments, the controller's responses to reject known disturbances were monitored. Known disturbances were consumption rate changes in

controlled and uncontrolled nodes (nodes 2 to 12). Since the distances between uncontrolled nodes and controlled nodes were not much, the closed-loop responses were approximately identical and responses of uncontrolled nodes were not shown. To decide which performance was the best, the Integral of the Square of the Error (ISE) for all nodes and the consumption of energy in compressor station were considered. In addition for safe gas distribution a minimum pressure must be guaranteed at all nodes. This minimum was selected equal to 590 bar. This means that the pressure of all nodes in all times must be greater than 590 bar.

In the first experiment, an increase of 60% was applied to the consumption rate of node 3 by a dynamic with the time constant of 1.38 hour. Figure 5 shows the controllers responses. As can be seen, the MPC controller rejected this disturbance quite effectively. Simple PI and selective PI had 9600% and 5700% more ISE than MPC, respectively. They also consumed more energy in compressor station, which was 105% for simple PI and 62% for selective PI more than the required energy for MPC. In addition, the overshoot, undershoot and the settling time in the case of using MPC is obviously lower than other schemes.

**Figure 5.** Node 3 Consumption Increased by 60%

In the second experiment, an increase of 60% was applied to the consumption rate of node 7 by a dynamic with the time constant of 1.38 hour. Figure 6 shows the controllers responses. As can be seen, the MPC controller has slightly better performance than simple PI controller, the performance of selective PI controller was the worst case. Simple PI and selective PI had 37% and 209% more ISE than MPC, respectively. They also consumed more energy in compressor station which was 3% for simple PI and 20% for selective PI more than required the energy for MPC. In addition, the overshoot and undershoot in the case of using MPC was obviously lower than other schemes, but the settling time was almost higher than other schemes.

In the third experiment, an increase of 30% was applied to the outlet flow rate of gas pipeline (node 13) by a dynamic with the time constant of 1.38 hour. This node was at the end of pipeline and its flow rate was relatively high (see Table 2). Figure 7 shows the controllers responses. As can be seen, in this experiment the selective PI had better performance than other schemes. Unlike the previous two experiments, the overshoot, undershoot, and the settling time in the case of using MPC were obviously higher than other schemes. This means that the selected structure and constraints for MPC were not suitable for this huge disturbance.

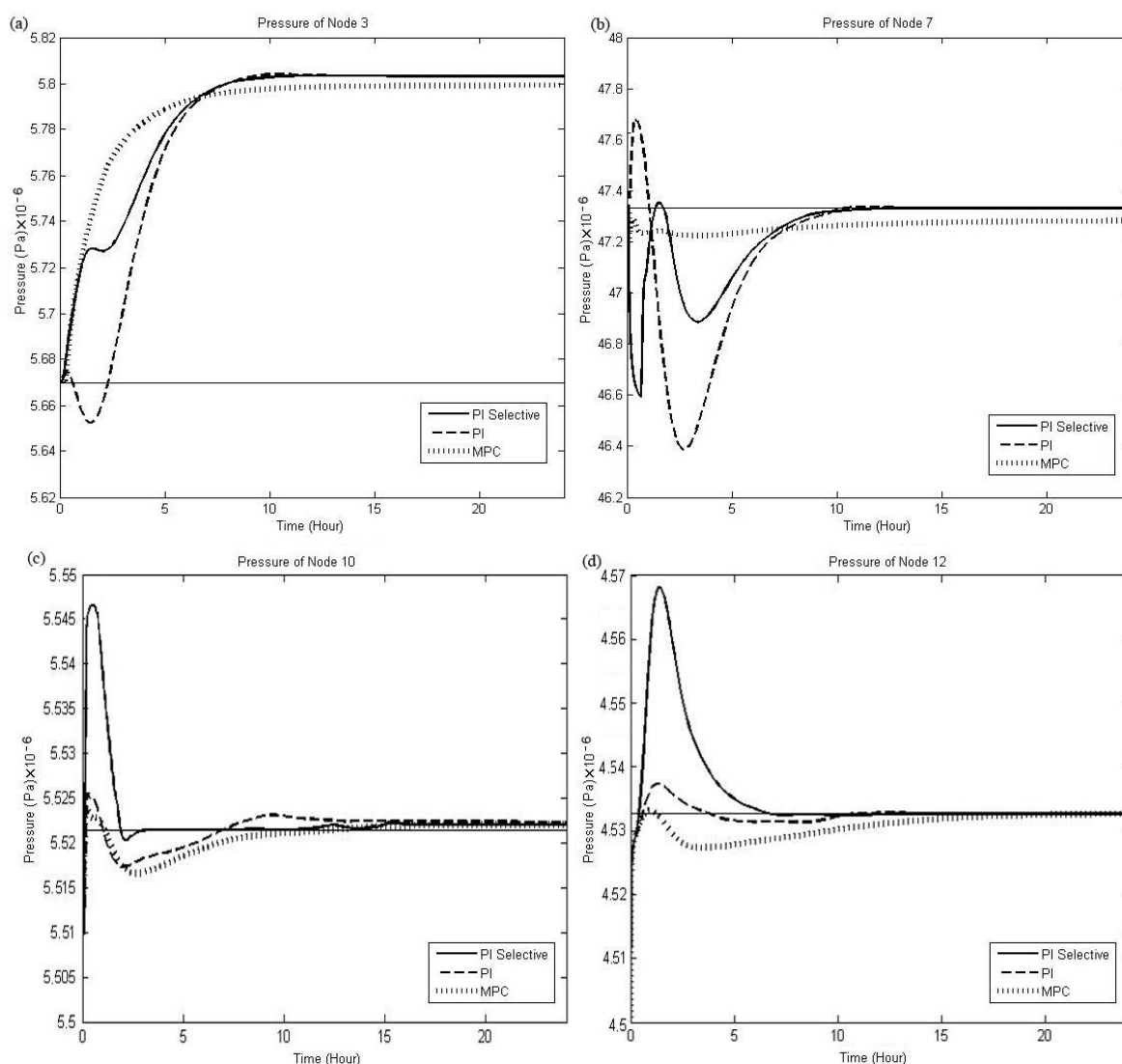


Figure 6. Node 7 Consumption Increased by 60%

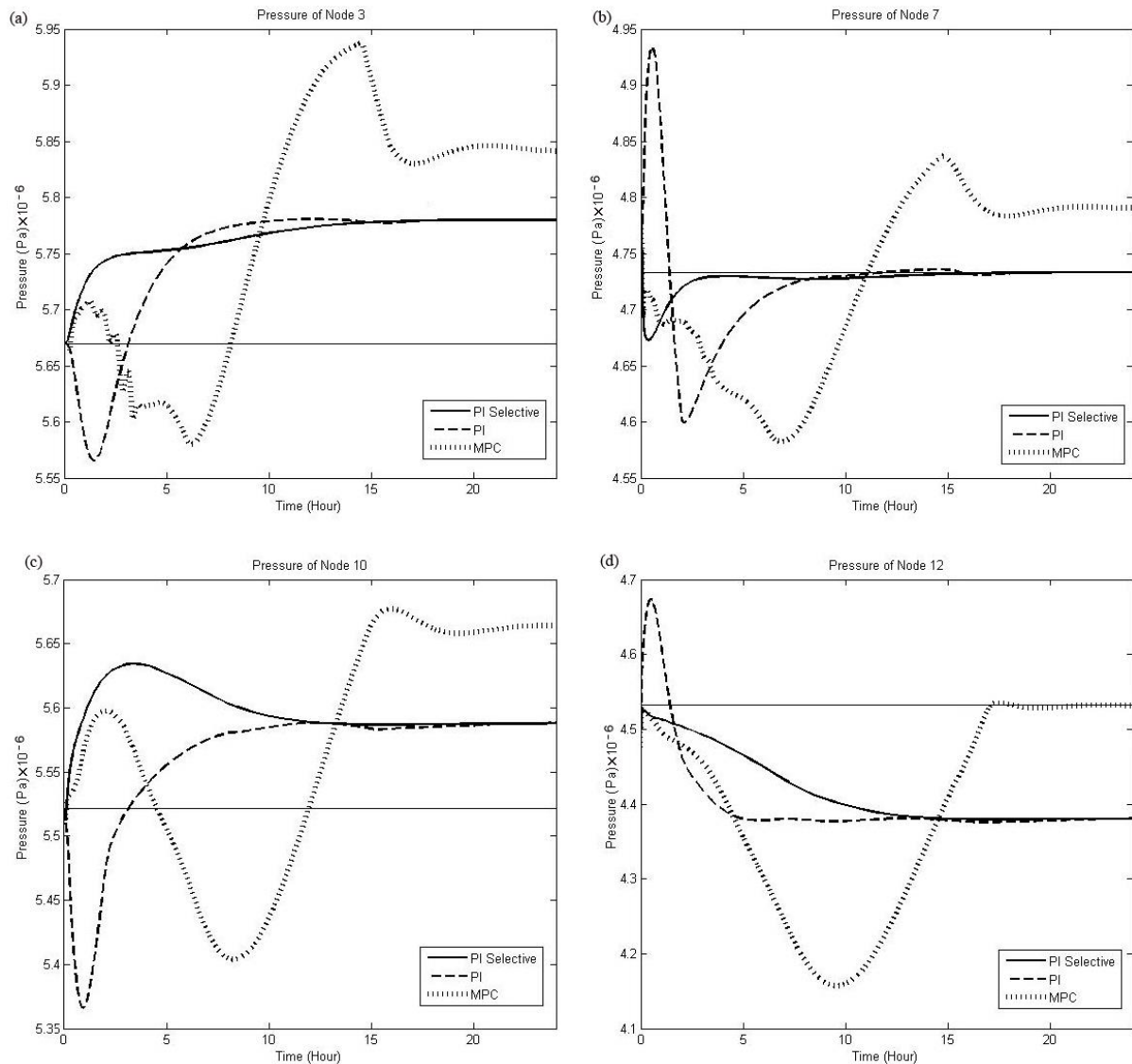


Figure 7. Node 13 Consumption Increased by 30%

5. Conclusions

Safety and profitability are necessary for the effective control of large scale natural gas transport pipeline. Although pipeline networks were a characteristic feature of the gas transport industry, the application of advanced control to such a system was not common. In this paper, a dynamic model based on continuity, momentum, and energy balances were selected for gas pipeline network from Khangiran refinery to Farooj compressor station (in Iran). A comparison between the simulated and actual pressures at steady state conditions did not show a considerable difference. Next, three controller schemes, MPC, simple PI, and selective PI were applied to the simulated pipeline. All schemes tried to control the pressure and supply consumers in an economic way without violating the physical constraints of production and distribution

facilities. The withdrawal rate changes of controlled and uncontrolled nodes showed that the MPC scheme had better performances than other schemes in disturbance rejecting of all nodes except the last node (outlet flow rate of gas pipeline). However, it should be noticed that the MPC controller had a more complicated structure and difficult design procedure than PI controllers.

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